Title	LABOUR-BIOPHYSICAL EXTENSION OF HASSELBALCH-HENDERSON'S EQUATION.
Author(s)	TAKANO, senseki
Citation	茨城大学文理学部紀要, 自然科学(8): 49-54
Issue Date	1958-02
URL	http://hdl.handle.net/10109/5375
Rights	

茨城大學文理學部紀要(自然科學)第8号 1958
Bull. Fac. Lib. Arts, Ibaraki Univ., Nat. Sci., No. 8, 1958

LABOUR-BIOPHYSICAL EXTENSION OF HASSELBALCH-HENDERSON'S EQUATION.

Senseki TAKANO*

高野千石：ハツセルバルハ・ヘンダースンの方程式の労働生物理学への拡張

1) Introduction.

I have already reported the fact[1] that a remarkable linear correlation was found between the fatigue index which was measured by the cupriferrocyanide colloidal reaction [2-5] and the similar index of the ZAMBRINI's reaction.[6] This fact shows that the mechanisms of both reactions are directly connected with the acid-alkali equelibrium of blood system, and obviously with the acid formation in the working tissues by glycolysis on muscular contraction. Hence, we can easily conclude that these reactions will open up the possibilities of quantitative estimation of the constants of acid-alkali equelibrium in the blood system, if we make an adequate unification to the various theories which were deduced by HASSELBALCH[7] and HENDERSON,[8] ZAMBRINI and WATANABE, and me myself. The question of acid-alkali equelibrium of the blood system contains many important physiological functions of the maintenance of life, and especially of the fatigue induced by muscular contraction.

Thus, if we settle the theoretical relations in this unknown field, the substance of the cupriferrocyanide colloidal reaction will be thoroughly understood in the various cases of labour-physiology.

2) Theoretical treatment.

Now, as we already reported, the saliva pH is expressed by the next formula[9] :

$$(pH)_S = \alpha z + \beta \quad \cdots\cdots (1)$$

where $(pH)_S$ denotes the hydrogen-ion exponent of saliva, z is the fatigue index of the ZAMBRINI's reaction and α, β are both constants.

The grounds of the formation of this formula are easy to be comprehended from the mechanism of that reaction. The general reagents of the reaction are 1. 2-oxyanthraquinone and 1. 2. 6.-oxyanthraquinone, and these are mere indicators of saliva $(pH)_S$ and the fatigue index z is graded in proportion to the saliva pH.

On the other hand, according to the WATANABE's research,[10] the $(pH)_S$ correlates with the VAN-SLYKE's alkali reserve in volume percent (V) in blood by the next

* Chemical Institute, Faculty of Liberal Arts, Ibaraki University.

formula:

$$V = \varepsilon + \{\gamma(pH)_S - \delta\} \quad \cdots\cdots(2)$$

where γ, δ and ε are constants. These numerical expressions are $\gamma = 12.5$, $\delta = 6.8$, $\varepsilon = 46.0$ respectively and these are already discussed in our previous paper.[11]

According to the studies of HASSELBALCH and HENDERSON, the hydrogen ion concentration of blood $(H^+)_b$ is expressed by the next formula:

$$(H^+)_b = k_1 \frac{[H_2CO_3]}{[BHCO_3]} \quad \cdots\cdots(3)$$

This is the original formula of HASSELBALCH - HENDERSON's equation. Now we put the VAN-SLYKE's alkali reserve of blood in volume percent as follows:

$$V = \lambda [BHCO_3] \quad \cdots\cdots(4)$$

where λ is a proportional constant, and so we have:

$$(H^+)_b = \frac{k_1 \lambda [H_2CO_3]}{V} \quad \cdots\cdots(5)$$

or

$$\left.\begin{aligned} (pH)_b &= pk_1' + log \frac{V}{[H_2CO_3]} \\ pk_1' &= -log\, k_1 \lambda \end{aligned}\right\} \quad \cdots\cdots(6)$$

The formula (6) is the special form of HASSELBALCH - HENDERSON's equation represented by the VAN-SLYKE's alkali reserve (V) of blood.

From equation (2) and equation (6), we have:

$$(pH)_b = pk_1' + log \frac{\{\gamma (pH)_S - \delta\gamma + \varepsilon\}}{[H_2CO_3]} \quad \cdots\cdots(7)$$

by putting

$$\varepsilon - \gamma\delta = \varepsilon'$$

then

$$(pH)_b = pk_1' + log \{\gamma (pH)_S + \varepsilon'\} - log[H_2CO_3] \quad \cdots(8)$$

From equation (8) and equation (2) we have

$$(pH)_b = pk_1' + log\{\gamma(\alpha z + \beta) + \varepsilon'\} - log[H_2CO_3] \quad \cdots(9)$$

or putting $\beta\gamma + \varepsilon' = \varepsilon''$, then

$$(pH)_b = pk_1' + log \left(1 + \frac{\alpha\gamma z}{\varepsilon''}\right) + log \frac{\varepsilon''}{[H_2CO_3]} \quad \cdots(10)$$

The value of $\alpha\gamma z/\varepsilon''$ culculated from the following values $\alpha = 0.2$, $\beta = 5.2$, $\gamma = 12.5$, $\delta = 6.8$ and $\varepsilon = 46.0$ respectively, is about $0.096153 \ldots z$.

This value of z is ordinarily smaller than 10.0, excepting in the non-fatigued state. Thus,

$$\alpha\gamma z/\varepsilon'' < 1,$$

so we can expand the formula (10) with the approximation of the second term, on taking account of the accuaracy of the ordinary measurements of pH in blood. We have:

$$(pH)_b \fallingdotseq pk_1' + \frac{1}{2{\cdot}302} \left\{ \frac{\alpha\gamma z}{\varepsilon''} - \frac{\alpha^2 \gamma^2 z^2}{2\varepsilon''^2} \right\} + log \frac{\varepsilon''}{[H_2CO_3]} \quad \cdots(11)$$

This is the new form of HASSLBALCH-HENDERSON's equation represented by the ZAMBRINI's index.

Now, we have already reported by the experiments the relation of both indexes of cupriferrocyanide colloidal reaction and ZAMBRINI's reaction. These results are sumarized as follows:

$$z = \mu - \nu N \quad \cdots\cdots(12)$$

where μ and ν are some constants, and N, the fatigue index of cupriferrocyanide colloidal reaction, is culculated from the titrated volume of urine by the next formula[13]:

$$N = \left(\frac{4}{x}\right)^{0.87} \quad \cdots\cdots(13)$$

By the substitution of z in the formula (12), we have:

$$(pH)_b = pk_1' + \log\left\{1 + \frac{\alpha\gamma}{\varepsilon''}(\mu - \nu N)\right\} + \log\frac{\varepsilon''}{[H_2CO_3]} \quad \cdots(14)$$

or approximately

$$(pH)_b = pk_1' + \frac{\alpha\gamma(\mu-\nu N)}{2{\cdot}302\,\varepsilon''}\left\{1 - \frac{\alpha\gamma}{2\varepsilon''}(\mu-\nu N)\right\} + \log\frac{\varepsilon''}{[H_2CO_3]} \quad \cdots(15)$$

Likely to the formula (11), this is also the new form of HASSELBALCH-HENDERSON's equation represented by the fatigue index of the cupriferrocyanide colloidal reaction.

Although the numerical treatment of the various constants is omitted here, these equations of (11), (12) and (14) all contain in them the fatigue index z or N, so we can analyse the presumed relation of pH of blood and free carbonic acid activity in blood at any state of fatigue. By the substitution of the following constants which were already determined[1] experimentally in the previous paper, we have:

Table of Constants.

$$\varepsilon' = \varepsilon - \gamma\delta = -39.00, \quad \mu = 13.70, \quad \nu = 1.87, \quad \varepsilon'' = \beta\gamma + \varepsilon' = 26.00$$

$$(pH)_b = pk_1' + \log(1 + 0.09615z) + \log\frac{26}{[H_2CO_3]} \quad \cdots(10')$$

$$(pH)_b = pk_1' + 0.04174z + 0.00201z^2 + \log\frac{26}{[H_2CO_3]} \quad \cdots\cdots(11')$$

$$(pH)_b = pk_1' + \log(2.3165 - 0.1797N) + \log\frac{26}{[H_2CO_3]} \quad \cdots(14')$$

$$(pH)_b = pk_1' + (0.5719 - 0.07806N) \times (0.3417 + 0.08985N) + \log\frac{26}{[H_2CO_3]} \quad \cdots(15')$$

If we take the secondary approximation, we have:

$$(pH)_b = pk_1' + 0.04174z + \log\frac{26}{[H_2CO_3]} \quad \cdots\cdots(16)$$

$$(pH)_b = pk_1' + 0.5719 - 0.07806N + \log\frac{26}{[H_2CO_3]} \quad \cdots\cdots(17)$$

The measurable and significant order of blood pH is about two or three terms below zero, so in practical treatments the culculation of the above formulas (16) and (17) is good for use. On the other hand, the value pk_1' is estimated about 6.1 by HENDERSON, hence by substituting this value in the formulas (16) and (17) we have:

$$(pH)_b = 6.10 + 0.04174z + log \frac{26}{[H_2CO_3]} \cdots\cdots\cdots\cdots(16')$$

$$(pH)_b = 6.6719 - 0.07806N + log \frac{26}{[H_2CO_3]} \cdots\cdots\cdots(17')$$

3) Discussion and Culculation.

Culculating the free carbonic acid in blood in volume percent from the equation (17) on the various values of blood pH, we obtain the next table.

Table 2

pH of blood,	"a" in eq. (19)
6.9	−0.22808
7.0	−0.32808
7.1	−0.42808
7.2	−0.52808
7.3	−0.62808
7.4	−0.72808
7.5	−0.82808

where the parameter "a" denotes:

$$a = 6.6719 - (pH)_b \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(18)$$

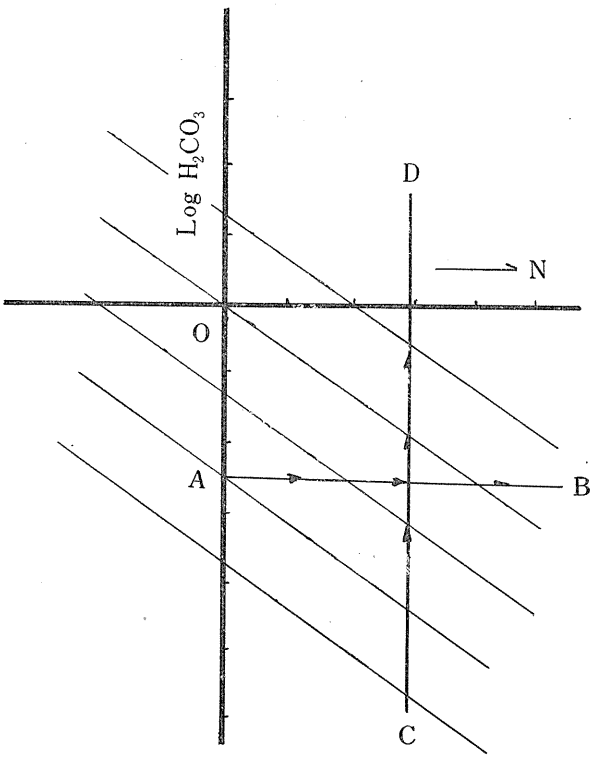
$$log \frac{[H_2CO_3]}{26} = a - 0.07806N \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(19)$$

The graph (I) shows the relation between the free carbonic acid activity of blood and the fatigue index N which is defined by the formula (13). We can easily appreciate the condition of blood in the real muscular work. When work is done, and the final products of metabolite are increased in the blood, the $log \frac{[H_2CO_3]}{26}$ at constant $(pH)_b$ decreases along the paralleled lines in proportion to the increase of fatigue index N.

If the gas exchange in the lungs is kept constant, then the blood pH decreases along the line $\overrightarrow{AB}$, according as N increases. So we may conclude that the increase of fatigue index N means the generation of acidocis when the gas exchange is kept constant.

But in the real muscular work, the gas exchange rate is generally promoted, so the above mentioned consideration is only an ideal case. On the contrary, if the gas exchange of the lungs is extremely interrupted, even the slightly loaded muscular work would cause the considerable increase of fatigue index N. Such

Graph I.



cases were often observed experimentally in the case of tuberclosis patients[14)15)]. These results were also traced by KUDOO[16)17)] and his co-workers, and they recognized the presence of the above mentioned cases.

Generally speaking, it is all right to think that the tension of free carbonic acid in the artery blood is equal to the tension of the gas in the lungs. But this assumption is not correct when the gas exchange of the lungs is interrupted, and, in such a case, the abnormality of the increasing of the index of cupriferrocyanide colloidal reaction will have something of clinical importance.

The various experiments on the reaction in the field of tuberclosis may have been undertakcn according to the above mentioned category. Such an abnormality of increasing fatigue index of the reaction due to the above mentioned gas exchange interruption in the lungs is not only seen in tuberclosis, but also will be observed in cardioic dyspnea.

Literature.

1) S. TAKANO, Bull. Fac. Lib. Arts, Ibaraki University, 2, (1952), 91-93.
2) S. TAKANO, Science (Japan), 20, (1950), 478.
3) S. TAKANO, Science (Japan), 21, (1951), 297.
4) S. TAKANO, Bull. Fac. Lib. Arts, Ibaraki University, 5, (1955), 12.
5) S. TAKANO, J. of Med. Prog., 39, (1952). and 38, (1951).
6) T. OGATA and I. WATANABE, Proc. Imp. Acad., (Tokyo), 8, (1940), 426.
7) HASSELBALCH, K. A, Biochem. Zeitschr., (1916), 112.
8) HENDERSON, I. J. Ergeb. Physiol., 8. (1909), 254.
9) S. TAKANO, Bull. Fac. Lib. Arts, Ibaraki University, 2, (1951), 91-93.
10) T. OGATA and I. WATANABE, ibid., (6).

11) S. TAKANO, ibid. (1).
12) ibid, (7) and (8).
13) S. TAKANO, Science (Japan), 20, (1950) 478.
14) S. TAKANO, Science (Japan), 17, (1947) 35.
15) S. TAKANO, Science (Japan), 17, (1947) 177.
16) B. KUDOO, Fukushima J. Med. Sci., 1. (1954) 43-49. (Dissertation)
17) B. KUDOO, Fukushima J. Med. Sci., 1. (1954) 71-79. (Dissertation)