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Some Theorems for Semigroups

RYÜKI MATSUDA*

Dedicated to Professor Hasumi
on his retirement from Ibaraki University

In Zariski-Samuel[10], Theorems 30 and 31 of Ch.IV, and Theorems 7 and 14 of Ch.V are, without saying, important theorems in commutative ring theory. And so is Theorem A of Appendix 2 of Gilmer[2]. The aim of this paper is to prove these theorems for semigroups. We will prove all these theorems for semigroups except [10,Ch.IV,Theorem 31]. We will prove [10,Ch.IV,Theorem 31] for semigroups of dimensions 1 and 2.

A subsemigroup $A_0$ of a torsion-free abelian (additive) group is called a $g$-monoid. Explicitly, semigroups we mentioned above mean $g$-monoids. Many propositions for commutative rings are known to hold for $g$-monoids. The author conjectures that almost all propositions in multiplicative ideal theory hold for $g$-monoids (cf.[5]).

Let $Z_0$ be the non-negative integers. A $g$-monoid $S$ of the form $\sum_i^n Z_0 s_i$ for a finite number of elements $s_1, \ldots, s_n$ of $S$ is called a finitely generated semigroup. Let $P$ be a prime ideal of a $g$-monoid $S$. Then the height $ht(P)$ of $P$ is naturally defined. If there exist no prime ideals $\notin P$, then $ht(P) = 1$. The semigroup ring of a $g$-monoid $S$ over a commutative ring $R$ is denoted by $R[X;S]$, where $X$ is a symbol.

LEMMA 1. Let $I$ be a proper ideal in a finitely generated semigroup $S$. If $I$ is $r$-generated, that is, generated by $r$ elements of $S$, then every prime ideal $P$ minimal among containing $I$ has height at most $r$.

PROOF. Let $k$ be a field. Then the semigroup ring $k[X;S]$ of $S$ over $k$ is a Noetherian ring, and $Pk[X;S]$ is a prime ideal minimal among containing $I k[X;S]$. Therefore $ht(Pk[X;S]) \leq r$. Hence $ht(P) \leq r$.

If every ideal of a $g$-monoid $S$ is finitely generated, $S$ is called a Noetherian semigroup.

LEMMA 2. Let $S$ be a Noetherian semigroup such that the unit group $H$ of $S$ is a free abelian group. Let $I$ be a proper ideal in $S$ generated by $r$ elements $a_1, \ldots, a_r$. If the maximal ideal $M$ of $S$ is a prime ideal minimal among containing $I$, then $ht(M) \leq r$.

PROOF. Let $\{u_\lambda \mid \Lambda\}$ be a free generators of $H$. By Lemma 1, we may assume that $|\Lambda| = \infty$. Let $p_1, \ldots, p_l$ be the set of non-associated irreducible

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elements of $S$. The quotient group $G$ of $S$ equals to $\sum_{\lambda} u_{\lambda} + \sum_i Zp_i$. There exist a subset $\Lambda_1$ and a finite subset $\{\lambda_1, \ldots, \lambda_n\}$ of $\Lambda$ such that $G = \sum_{\lambda} u_{\lambda} \oplus (\sum_i Zu_{\lambda_i} + \sum_i Zp_i)$. Let $k$ be a field. $Mk[X; S]$ is a prime ideal minimal among containing $Ik[X; S]$. Set $S_1 = \sum_i Zu_{\lambda_i} + \sum_i Zp_i$, $D = k[X; \sum_{\lambda} u_{\lambda}]$, and let $K$ be the quotient field $q(D)$ of $D$. We see that $Mk[X; S] = D[X; S_1]$, and $MK[X; S]$ is a prime ideal minimal among containing $(a_1, \ldots, a_r) K[X; S_1]$. It follows that $\text{ht}(MK[X; S_1]) \leq r$. Hence $\text{ht}(MK[X; S]) \leq r$. Therefore $\text{ht}(M) \leq r$.

**Theorem 1.** Let $I$ be a proper ideal of a Noetherian semigroup $S$. If $I$ is generated by $r$ elements, then every prime ideal minimal among containing $I$ has height at most $r$.

**Proof.** We may assume that $P$ equals to the maximal ideal $M$ of $S$. Let $\{u_{\lambda} \mid \Lambda\}$ be a maximal independent subset of the unit group $H$ of $S$. By Lemma 1, we may assume that $|\Lambda| = \infty$. Let $p_1, \ldots, p_r$ be the set of all non-associated irreducible elements of $S$. We may assume that $M$ is a prime ideal minimal among containing $(p_1, \ldots, p_r)$. Set $S_1 = \sum_{\Lambda} Zu_{\lambda} + \sum_i Zp_i$. Then $S$ is integral over $S_1$, and the maximal ideal $M_1$ of $S_1$ is a prime ideal minimal among containing $(p_1, \ldots, p_r) S_1$. By Lemma 2, $\text{ht}(M_1) \leq r$. Hence $\text{ht}(M) \leq r$.

Let $S$ be a $g$-monoid, and $Y$ a non-empty set. Assume that, for each $s \in S$ and $x \in Y$, an element $s + x \in Y$ is determined, and, for each $s_1$ and $s_2$ of $S$, $(s_1 + s_2) + x = s_1 + (s_2 + x)$, and $0 + x = x$. Then $Y$ is called an $S$-module.

**Theorem 2.** Let $S$ be an integrally closed semigroup, $G$ its quotient group, $L$ an extension torsion-free abelian group of $G$ with $(L : G) = n < \infty$, and $T$ the integral closure of $S$ in $L$. Then there exists a complete representative system $x_1, \ldots, x_n$ of $L$ modulo $G$ such that $T$ is contained in the $S$-module $\bigcup_i (S + xi)$.

**Proof.** Let $k$ be a field of characteristic 0. Then $k[X; S]$ is an integrally closed domain, the quotient field $F$ of $k[X; T]$ is a finite separable algebraic extension of the quotient field $K$ of $k[X; S]$, and $k[X; T]$ is the integral closure of $k[X; S]$ in $F$. Hence there exists a basis $\{\varphi_1, \ldots, \varphi_m\}$ of $F$ over $K$ such that $k[X; T]$ is contained in the $k[X; S]$-module $\sum_i k[X; S] \varphi_i$. Therefore there exist elements $\alpha_1, \ldots, \alpha_l$ of $L$ such that $T \subset \bigcup_i (S + \alpha_i)$. Let $\{x_1, \ldots, x_n\}$ be a complete representative system of $L$ modulo $G$. We may assume that $\{x_1, \ldots, x_n\} \subset \{\alpha_1, \ldots, \alpha_l\}$. For each $i$, $\alpha_i$ is contained in $x_j + G$ for some $j$. Therefore $\alpha_i - x_j + a_i \in S$ for some $a_i \in S$. Put $s = a_1 + \ldots + a_l$. Then we see that $S + \alpha_i \subset S + x_j - s$. It follows that $T \subset \bigcup_i (S + x_i - s)$.

**Proposition 1.** Let $S$ be a Noetherian semigroup, and $Y$ be a finitely generated $S$-module. Then $Y$ satisfies the a.c.c. on submodules.

**Proof.** It is sufficient to consider the case where $Y$ is a cyclic module $S+x$. Suppose that $Y_1 \subset Y_2 \subset \cdots$ be an ascending chain of submodules of $Y$. For each $i$, let $A_i = \{a \in S \mid a + x \in Y_i\}$. Then $A_1 \subset A_2 \subset \cdots$ is an ascending chain of ideals of $S$. There is an $n$ such that $A_i = A_n$ for each $i > n$. Then $Y_i = Y_n$ for each $i > n$.

Theorem 2 and Proposition 1 imply the following,
Corollary 1. The assumptions being the same as in Theorem 2, let us furthermore assume that $S$ is Noetherian. Then $T$ is a finitely generated $S$-module and is a Noetherian semigroup.

Lemma 3. Let $S$ be a Noetherian semigroup, $A$ and $B$ be two ideals of $S$ such that $A \neq S$. Then $A = A :_S B$ if and only if $B$ is contained in no prime ideal of $A$.

The proof is an analogy for semigroups of the proof of [10, Ch.IV, Theorem 11].

Lemma 4. Let $S$ be a Noetherian integrally closed semigroup, and $P$ be a maximal ideal of $S$. If $P$ is a prime ideal of a principal ideal $(y)$, then $P$ is a principal ideal.

Proof. We have $(y) :_S P \subseteq (y)$ by Lemma 3. Choose an element $x$ of $(y) :_S P$ such that $x \notin (y)$. Then $x - y \notin P$, and $x - y \notin S$. Suppose that $P + P^{-1} \subseteq S$. Then we have $P = P + P^{-1}$. Set $P = (x_1, \ldots, x_n)$, and put $z = x - y$. Then $z + x_i \in P$ for each $i$. We may assume that $z + x_1 = x_2 + r_1, z + x_2 = x_3 + r_2, \ldots, z + x_k = x_1 + r_k$ for some $k \geq 1$ and for some elements $r_1, \ldots, r_k$ of $S$. Then we have $kz = r_1 + \cdots + r_k \in S$. Hence $x - y = z \in S$; a contradiction. We have proved that $P + P^{-1} = S$. There exist $p \in P$ and $q \in P^{-1}$ such that $p + q = 0$. Then we have $P = (p)$.

Lemma 5. Let $A$ be a proper ideal of a $g$-monoid $S$ admitting an irredundant primary representation $A = \bigcap_1^n Q_i$. Let $T$ be an additive system of $S$, and suppose that, for $1 \leq i \leq r$, we have $Q_i \cap T = \emptyset$, and that, for $r + 1 \leq j \leq n$, we have $Q_j \cap T \neq \emptyset$. Then $AS_T = \bigcap_1^n (Q_iS_T)$ is an irredundant primary representation of $AS_T$.

The proof is straightforward.

Theorem 3. In an integrally closed Noetherian semigroup $S$, each prime ideal $P$ of any proper principal ideal $(y)$ has height 1.

Proof. $SP$ is a Noetherian integrally closed semigroup, and $PS_P$ is a maximal ideal of $SP$ which is a prime ideal of a principal ideal $ySP$ by Lemma 5. Lemma 4 implies that $PS_P$ is a principal ideal. Therefore $PS_P$ has height 1. Hence $P$ has height 1.

Let $P$ be a prime ideal of a $g$-monoid $S$. If $P$ is the only $P$-primary ideal of $S$, then $P$ is called unbranched.

Theorem 4. Let $V$ be a nontrivial valuation semigroup with quotient group $L$, and suppose that $V$ is of the form $G \cup M$, where $G$ is a group and $M$ is the maximal ideal of $V$. Let $S$ be a $g$-monoid which is a proper subsemigroup of $G$, and let $S_1 = S \cup M$.

(1) $S_1$ is a $g$-monoid and $M$ is the conductor of $S_1$ in $V$. Therefore, $S_1$ and $V$ have the same complete integral closure. In particular, $S_1$ is not completely integrally closed.
(2) The integral closure of $S_1$ is $\bar{S} \cup M$, where $\bar{S}$ is the integral closure of $S$ in $G$.

(3) Each ideal of $S_1$ compares with $M$ under $\subset$.

(4) The set of ideals of $S_1$ containing $M$ is $M$ and $\{A_\alpha \cup M \mid \alpha\}$ where $\{A_\alpha \mid \alpha\}$ is the set of ideals of $S$. If $A_{\alpha_1} \cup M = A_{\alpha_2} \cup M$, then $A_{\alpha_1} = A_{\alpha_2}$. $M$ is a prime ideal of $S_1$. Further, $A_\alpha$ is maximal, prime, or $P_\alpha$-primary in $S$ if and only if $A_\alpha \cup M$ is, respectively, maximal, prime, or $(P_\alpha \cup M)$-primary in $S_1$. If $T_\alpha$ is a generating set for $A_\alpha$ as an ideal of $S$, then $T_\alpha$ is also a generating set for $A_\alpha \cup M$ as an ideal of $S_1$.

(5) If $Q$ is $P$-primary in $S_1$, where $P \subsetneq M$, then $Q$ and $P$ are ideals of $V$ and $Q$ is $P$-primary in $V$. If $M$, as an ideal of $V$, is unbranched, then $M$ is also unbranched as an ideal of $S_1$.

(6) $\dim S_1 = \dim S + \dim V$.

(7) If $N$ is an additive system in $S$, then $(S_1)_N = S_N \cup M$. If $P$ is prime in $S_1$ and if $P \subsetneq M$, then $(S_1)_P = V_P$ so that $(S_1)_P$ is a valuation semigroup.

(8) $S_1$ is a valuation semigroup on $L$ if and only if $S$ is a valuation semigroup on $G$.

(9) The valuative dimension of $S_1$ is equal to $k + \dim V$, where $k$ is the maximal dimension of a valuation semigroup on $G$ containing $S$. ($k$ may be infinite.)

(10) The finitely generated ideals of $S_1$ which properly contain $M$ are those of the form $A_\alpha \cup M$, where $A_\alpha$ is a finitely generated ideal of $S$. Any finitely generated ideal $A$ of $S_1$ contained in $M$ can be obtained as follows: let $W$ be a finitely generated $S$-submodule of $G$, let $m \in M$, and set $A = (W+m) \cup (M+m)$.

(11) $S_1$ is Noetherian if and only if $V$ is Noetherian, $S$ is a group, and $(G:S) < \infty$.

Almost all of the proof of Theorem 4 is an analogy of the proof of [2, Appendix 2, Theorem A]. The sufficiency of (11): $V$ is the integral closure of $S_1$. By the Mori-Nagata theorem for semigroups [6], $V$ is Noetherian.

Let $S$ be a 1-dimensional Noetherian semigroup, and $M$ be a maximal ideal of $S$. Then, clearly, $M$ is a prime ideal minimal among containing a 1-generated ideal of $S$.

Let $S$ be a g-monoid, and $x_1, \ldots, x_n$ be elements of an extension semigroup of $S$. Then $S + \sum_i \mathbb{Z}_0 x_i$ is called the semigroup generated by $x_1, \ldots, x_n$ over $S$, and is denoted by $S[x_1, \ldots, x_n]$.

**Theorem 5.** Let $S$ be a 2-dimensional Noetherian semigroup, and $M$ be its maximal ideal. Then $M$ is a prime ideal minimal among containing a 2-generated ideal of $S$.

**Proof.** Assume that $M$ is $l$-generated. Assume that $l \geq 3$, and that our assertion holds for $l - 1$. Suppose that $M$ is not a prime ideal minimal among containing a 2-generated ideal of $S$. We will derive a contradiction.

Let $M = (x_1, \ldots, x_l)$. We may assume that $\{x_1, \ldots, x_l\}$ is a complete representative system of irreducible elements of $S$. Let $H$ be the unit group of $S$. 

At first, we will show that, for each $j$, $x_j$ is not integral over the semigroup $H[x_{\alpha} \mid \alpha \neq j]$ generated by $\{x_{\alpha} \mid 1 \leq \alpha \leq l, \alpha \neq j\}$ over $H$. Thus, suppose, for instance, that $x_l$ is integral over $H[x_{x_1, \ldots, x_{l-1}}]$. Then $S_1 = H[x_{x_1, \ldots, x_{l-1}}]$ is a 2-dimensional Noetherian semigroup. And the maximal ideal $M_1$ of $S_1$ is generated by $x_1, \ldots, x_{l-1}$. Therefore $M_1$ is a prime ideal minimal among containing a 2-generated ideal $(y_1, y_2)S_1$ of $S_1$. Since $S$ is integral over $S_1$, we see that $M$ is a prime ideal minimal among containing $(y_1, y_2)S$; a contradiction.

Next, let $y_1, \ldots, y_{l-1}$ be $l-1$ distinct members in $\{x_1, \ldots, x_l\}$. We will show that $(y_1, \ldots, y_{l-1})$ is a ht 1 prime ideal of $S$. Thus, suppose, for instance, that the ideal $I = (x_1, \ldots, x_{l-1})$ is not prime. Then any prime ideal which contains $I$ contains $x_l$. Therefore $x_l$ is integral over $H[x_{x_1, \ldots, x_{l-1}}]$; a contradiction.

Next, we will show that, for each $1 < i < j < k < l$, there exist natural numbers $a_i$ and $a_j$ such that $a_ix_i + a_jx_j$ belongs to the semigroup $H[x_{\alpha} \mid 1 \leq \alpha \leq k, \alpha \neq i, j]$ generated by $\{x_{\alpha} \mid 1 \leq \alpha \leq k, \alpha \neq i, j\}$ over $H$. We depend on the induction on $k$ from $l$. Thus, assume that $k = l$. Let $P = (x_{\alpha} \mid \alpha \neq i)$ be the ideal of $S$ generated by $\{x_{\alpha} \mid 1 \leq \alpha \leq l, \alpha \neq i\}$. Then $P$ is a prime ideal of $S$, and $P_S$ is a 1-dimensional Noetherian semigroup. Hence $P_S P$ is a prime ideal minimal among containing the ideal $x_l S_P$. Therefore, for each $j$, $a_jx_j$ belongs to $x_l S_P$ for some natural number $a_j$. It follows that $a_ix_i + a_jx_j \in (x_l)$ for some $a_i \in \mathbb{Z}_0$. $x_l$ is not a unit of $S$, $x_j$ is not integral over $H[x_{\alpha} \mid \alpha \neq j]$, and $x_i$ is not integral over $H[x_{\alpha} \mid \alpha \neq i]$. Therefore $a_i > 0$, and moreover $a_ix_i + a_jx_j \in H[x_{\alpha} \mid \alpha \neq i, j]$.

Next, assume that $1 < i < j < k < l$, and that the assertion holds for $k + 1$. There exist natural numbers $a_i$ and $a_j$ such that $a_ix_i + a_jx_j \in H[x_{\alpha} \mid 1 \leq \alpha \leq k + 1, \alpha \neq i, j]$. Suppose that $a_ix_i + a_jx_j \in H[x_{\alpha} \mid 1 \leq \alpha \leq k, \alpha \neq i, j]$. Then we have $a_ix_i + a_jx_j = a_{k+1}x_{k+1} + \sum_{\alpha \neq i, j}^k b_{\alpha}x_{\alpha} + h_1(a_{k+1} > 0, \text{each } b_{\alpha} \geq 0, h_1 \in H)$, where $\sum_{\alpha \neq i, j}$ means that $\alpha$ ranges over $1 \leq \alpha \leq k$ with $\alpha \neq i, j$. Also we have $x_{k+1} = c_{k+1} + h_2(c_{k+1} > 0, d_{\beta} \geq 0, h_2 \in H)$. We may assume that $a_{k+1} = c_{k+1}$. Since $x_i$ is not integral over $H[x_{\alpha} \mid 1 \leq \alpha \leq k, \alpha \neq i]$, we have $a'_i x_i + a'_j x_j = \sum_{\alpha \neq i, j} b'_{\alpha} x_{\alpha} + h'(a'_i, a'_j > 0; b'_\alpha \geq 0, h' \in H)$.

Now if we apply the above result for $i = 2, j = 3, k = 3$, we have $a_2x_2 + a_3x_3 \in H[x_1]$ for natural numbers $a_2, a_3$. Therefore $x_1$ is integral over $H[x_2, x_3]$; a contradiction.

The motive of this paper, first, was to prove all such propositions in [10] and in [2] for g-monoids $S$ that have meaning for $S$. All these propositions in [10] except [10, Theorems 30 and 31 of Ch.IV, Theorems 7 and 14 of Ch.V] have been proved for g-monoids. However, since they have not been published, we will state them briefly, for convenience. A semigroup version of [3] is under preparation in [4]. The only such propositions in [2] that is not contained in [3] and has meaning for $S$ is [2, Appendix 2, Theorem A].

Let $Y$ be an $S$-module. Let $Y = Y_0 \supseteq Y_1 \supseteq \cdots \supseteq Y_r$ be submodules of $Y$. If, for each $i$, there exist no submodules $Y_i'$ such that $Y_i \supseteq Y_i' \supseteq Y_{i+1}$, then the chain of submodules of $Y$ is called a composition series of length $r$.

**Proposition 2.** If an $S$-module $Y$ has one composition series of length $r$,
then every composition series of \( Y \) has length \( r \).

The proof of Proposition 2 appears on [8].

**Proposition 3.** If \( S \) is a Noetherian semigroup, then so is any semigroup \( S[x_1, \ldots, x_n] \) generated by a finite number of elements \( x_1, \ldots, x_n \) over \( S \).

**Proof.** Let \( X_1, \ldots, X_n \) be indeterminates. Then [9, Theorem 69'] shows that \( S[X_1, \ldots, X_n] \) is a Noetherian semigroup. Let \( I \) be an ideal of \( S[x_1, \ldots, x_n] \). Set \( J = \{ s + \sum k_i X_i \mid \text{each } k_i \geq 0, s + \sum k_i x_i \in I \} \). Then \( J \) is a finitely generated ideal of \( S[X_1, \ldots, X_n] \). It follows that \( I \) is a finitely generated ideal of \( S[x_1, \ldots, x_n] \).

**Proposition 4.** Let \( S \) be a Noetherian semigroup. Then every proper ideal \( I \) of \( S \) admits an irredundant primary representation \( I = \bigcap_1^n Q_i \). The set of prime ideals \( \sqrt{Q_1}, \ldots, \sqrt{Q_n} \) are uniquely determined by \( I \).

The proof of Proposition 4 appears on [7, (1.1) Proposition] and [8].

**Proposition 5.** Let \( S \) be a Noetherian semigroup and \( I \) a proper ideal of \( S \). Then \( \bigcap_1^n nI = \emptyset \).

The proof of Proposition 5 appears on [9, Proposition 77].

**Proposition 6.** Let \( S \) be a g-monoid, \( T \) an extension semigroup which is integral over \( S \). If \( P \) is a prime ideal of \( S \), then there exists a unique prime ideal \( Q \) of \( T \) lying over \( P \).

**Proof.** Let \( \Sigma \) be the family of all ideals \( J \) of \( T \) so that \( J \cap (S - P) = \emptyset \). Then \( \Sigma \) is not empty. Let \( Q \) be a maximal member of \( \Sigma \) under the inclusion relation. Then \( Q \) is a prime ideal of \( T \), and lies over \( P \).

**Proposition 7.** Let \( S \) be a g-monoid, and \( S' \) an extension semigroup of \( S \) which is integral over \( S \). If \( P \) and \( Q \) are prime ideals in \( S \) such that \( Q \subseteq P \), and if \( P' \) is a prime ideal of \( S' \) lying over \( P \), then there exists a prime ideal \( Q' \) of \( S' \) uniquely, contained in \( P' \) and lying over \( Q \).

**Proof.** There exists a prime ideal \( Q' \) of \( S' \) uniquely which lies over \( Q \) by Proposition 6. Let \( x \in Q' \). We have \( nx \in S \) for some \( n > 0 \). Then \( nx \in Q \subseteq P \subseteq P' \). It follows that \( Q' \subseteq P' \).

**Proposition 8.** Let \( V \) be a discrete valuation semigroup of rank 1, and \( L \) an extension torsion-free abelian group of the quotient group \( G \) of \( V \) with \( (L : G) < \infty \). Then the integral closure \( W \) of \( V \) in \( L \) is a discrete valuation semigroup of rank 1.

For the proof, choose elements \( x_1, \ldots, x_n \) in \( W \) such that the quotient group of \( V[x_1, \ldots, x_n] \) is \( L \). \( V[x_1, \ldots, x_n] \) is a Noetherian semigroup by Proposition 3. Then \( W \) is a 1-dimensional Krull semigroup by [6]. Hence \( W \) is a discrete valuation semigroup of rank 1.

**Question.** If \( P \) is a prime ideal of height \( n \) in a Noetherian semigroup \( S \), then is \( P \) a prime ideal minimal among containing an \( n \)-generated ideal of \( S \)?
Some Theorems for Semigroups

References